

# Neutrino photoproduction on pseudo Nambu–Goldstone bosons

I. Alikhanov

Institute for Nuclear Research of the Russian Academy of Sciences, 60-th October Anniversary pr. 7a, Moscow 117312, Russia

E-mail: [ialspbu@gmail.com](mailto:ialspbu@gmail.com)

**Abstract.** Production of single neutrinos as well as neutrino–antineutrino pairs by photons interacting with pseudo Nambu–Goldstone bosons is studied within the Standard Model. The corresponding cross sections are found analytically. The energy loss due to neutrino emission in a thermal plasma of photons and pions (kaons) is calculated and some related implications for astrophysics are discussed. It is shown that the obtained neutrino emissivities may be significantly enhanced in dense matter due to in-medium modification of the total pion decay width.

**Keywords:** Nambu–Goldstone boson, pion decay, neutrino emission, stellar evolution

---

## Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Neutrino–antineutrino pair photoproduction on neutral pNGB</b>	<b>1</b>
<b>3</b>	<b>Single neutrino photoproduction on charged pNGB</b>	<b>4</b>
<b>4</b>	<b>Conclusions</b>	<b>5</b>
<b>A</b>	<b>The matrix element squared</b>	<b>6</b>

---

## 1 Introduction

Nambu–Goldstone bosons (often referred to as Goldstone bosons) appear necessarily in quantum field theories with spontaneously broken global continuous symmetries [1–3]. The bosons remain massless provided the symmetries are exact, acquiring masses only in the case of approximate symmetries. In the latter case they are called pseudo Nambu–Goldstone bosons (pNGB).

In nature, pNGB manifest themselves as the lightest pseudoscalar mesons from the SU(3) flavor octet – the pions [2, 4, 5]. This happens due to the spontaneous chiral symmetry breaking in quantum chromodynamics. The kaons may also be identified as pNGB [6].

PNGb modes appearing in dense matter formed inside astrophysical objects, such as core collapse supernovae and compact stars, could make a dramatic impact on the stellar thermal evolution. In 1965 Bahcall and Wolf demonstrated that a neutron star containing free pions in its interiors would cool much faster through neutrino emission in comparison with the conventional mechanisms – the modified Urca and bremsstrahlung neutrino processes [7]. Since then, the role of PNGb modes for compact star cooling attracts much attention [8–13].

In this paper production of neutrinos in interactions of photons with PNGb is studied within the Standard Model. Specifically, the neutrino emissivities of a star through the processes  $\gamma + \pi^0 \rightarrow \nu + \bar{\nu}$ ,  $\gamma + (\pi^+, K^+) \rightarrow e^+ + \nu_e$  and  $\gamma + (\pi^+, K^+) \rightarrow \mu^+ + \nu_\mu$  are calculated.

## 2 Neutrino–antineutrino pair photoproduction on neutral pNGB

The Standard Model accommodates the following interaction [14–17]:

$$\gamma + \pi^0 \rightarrow \nu_l + \bar{\nu}_l, \quad (2.1)$$

represented by the Feynman diagram in figure 1 ( $l = e, \mu, \tau$ ).

The corresponding matrix element is [14, 15, 17]

$$\mathcal{M} = -\frac{eG_F}{\sqrt{2}m_\pi} F_V \varepsilon_\mu \bar{u}(p'_\nu) \gamma_\alpha (1 - \gamma_5) v(p_\nu) \epsilon^{\mu\alpha\beta\lambda} q_\beta p_{\pi\lambda}. \quad (2.2)$$

Here  $e$  is the elementary electric charge,  $G_F$  is the Fermi coupling constant,  $\varepsilon_\mu$  denotes the photon polarization vector,  $p_\pi$ ,  $p_\nu$ ,  $p'_\nu$ , and  $q$  are the four-momenta of  $\pi^0$ , the final neutrinos and  $\gamma$ , respectively,  $F_V$  is the pion vector form factor.

Squaring (2.2) yields

$$\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{\alpha \pi G_F^2}{m_\pi^2} |F_V|^2 s (t^2 + u^2), \quad (2.3)$$

where  $\alpha$  is the fine structure constant,  $s = (p_\pi + q)^2$ ,  $t = (p_\pi - p_\nu)^2$  and  $u = (p_\pi - p_\nu)^2$  are the Mandelstam variables.

After the standard algebra one obtains the cross section of (2.1) for each neutrino flavor:

$$\sigma_\pi = \frac{\alpha G_F^2}{24 m_\pi^2} |F_V|^2 s^2 \left(1 - \frac{m_\pi^2}{s}\right). \quad (2.4)$$

Details of similar calculations can be found in [17]. Note that though the formfactor depends, in general, on the momentum transfer  $t$ , in the present analysis it is taken to be constant since we deal with reactions proceeding at conditions comparable to the case of the decay  $\pi_{e2\gamma}$  ( $t \sim m_\pi^2$ ) [18].

Consider matter containing photons and pions. If thermal equilibrium takes place at temperature  $T$  and the medium is transparent to the outgoing neutrinos (i.e. there is no Pauli blocking for the final state particles), the energy loss rate per unit volume due to emission of neutrinos of flavor  $l$  (the emissivity) via the reaction (2.1) is given by

$$Q_{\nu\bar{\nu}} = \frac{2}{(2\pi)^6} \int \frac{d^3\mathbf{k}_\gamma}{[\exp(\omega_\gamma/T) - 1]} \frac{d^3\mathbf{k}_\pi}{[\exp(\omega_\pi/T) - 1]} (\omega_\gamma + \omega_\pi) \sigma_\pi v_r, \quad (2.5)$$

where  $\omega_\gamma$  and  $\omega_\pi$  are the photon and pion energies, respectively,  $\mathbf{k}_\gamma$  and  $\mathbf{k}_\pi$  are their three-momenta,  $v_r$  is the relative velocity

$$v_r = \frac{\omega_\gamma \omega_\pi - \mathbf{k}_\gamma \cdot \mathbf{k}_\pi}{\omega_\gamma \omega_\pi}. \quad (2.6)$$

The pion vector form factor,  $F_V$ , is related via the Conserved Vector Current hypothesis (CVC) to the  $\pi^0 \rightarrow \gamma\gamma$  decay width  $\Gamma_{\pi^0 \rightarrow \gamma\gamma}$  by [19, 20]

$$|F_V|^2 = \frac{2\Gamma_{\pi^0 \rightarrow \gamma\gamma}}{\alpha^2 \pi m_\pi}. \quad (2.7)$$

Let us rewrite the cross section (2.4) by invoking the CVC:

$$\sigma_\pi = \frac{G_F^2}{6\alpha\pi m_\pi^3} \Gamma_{\pi^0 \rightarrow \gamma\gamma} (m_\pi^2 + 2\omega_\gamma \omega_\pi v_r) \omega_\gamma \omega_\pi v_r, \quad (2.8)$$

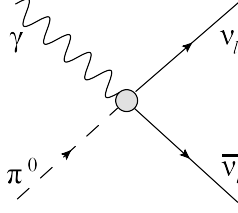
Note that  $s = m_\pi^2 + 2\omega_\gamma \omega_\pi v_r$ .

Calculation of the momentum space integrals in (2.5) taking into account (2.6) and (2.8) gives

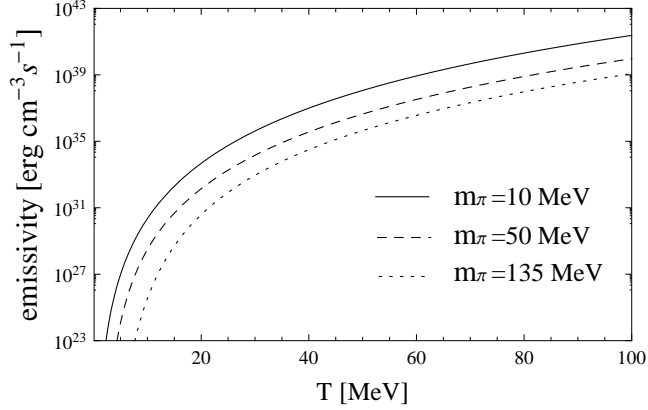
$$Q_{\nu\bar{\nu}} = \frac{G_F^2}{3780\alpha\pi^5 m_\pi^3} \Gamma_{\pi^0 \rightarrow \gamma\gamma} T^4 \mathcal{I}(m_\pi, T), \quad (2.9)$$

where

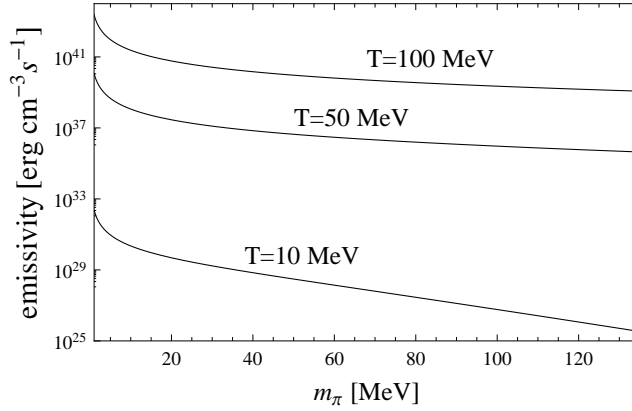
$$\mathcal{I}(m_\pi, T) = \int_{m_\pi}^{\infty} d\omega_\pi \frac{\sqrt{\omega_\pi^2 - m_\pi^2}}{\exp(\omega_\pi/T) - 1} [2520\zeta(5)T(12\omega_\pi^4 - 2m_\pi^2\omega_\pi^2 - m_\pi^4) - \pi^4\omega_\pi(80\pi^2 m_\pi^2 T^2 - 4\omega_\pi^2(7m_\pi^2 + 40\pi^2 T^2) + 7m_\pi^4)], \quad (2.10)$$



**Figure 1.** Feynman diagram for the process  $\gamma\pi^0 \rightarrow \nu_l\bar{\nu}_l$ .



**Figure 2.** The energy loss rate due to emission of neutrinos of flavor  $l$  (the emissivity) via the reaction  $\gamma\pi^0 \rightarrow \nu_l\bar{\nu}_l$  as a function of temperature.

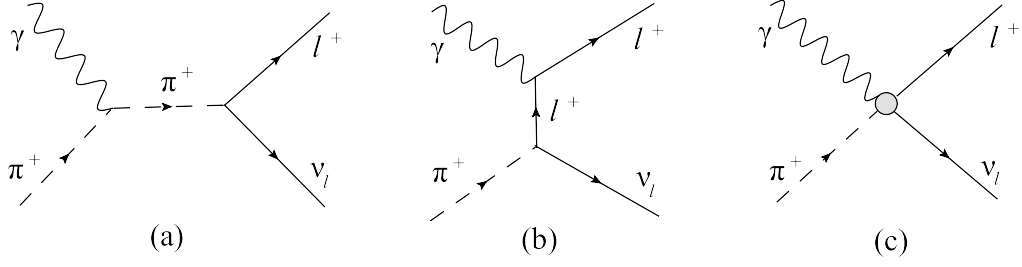


**Figure 3.** The energy loss rate due to emission of neutrinos of flavor  $l$  (the emissivity) via the reaction  $\gamma\pi^0 \rightarrow \nu_l\bar{\nu}_l$  as a function of the pion mass.

$\zeta(x)$  is the Riemann zeta-function ( $\zeta(5) = 1.037$ ).

The temperature dependence of the emissivity  $Q_{\nu\bar{\nu}}$  at three fixed values of the pion mass  $m_\pi$  is depicted in figure 2. Figure 3 shows  $Q_{\nu\bar{\nu}}$  as a function of  $m_\pi$ . The total energy loss can be easily found just by multiplying  $Q_{\nu\bar{\nu}}$  by the number of neutrino flavors.

Note that everywhere in the calculations presented in figures 2 and 3 the vacuum value of  $\Gamma_{\pi^0 \rightarrow \gamma\gamma}$  is used. Meanwhile, the pion decay width in the medium may be orders of magnitude larger than in vacuum [21]. The width at appropriate pion energies and momenta grows up to tens of MeV with the density [11] leading thus, according to (2.9), to a significant



**Figure 4.** Feynman diagrams for the process  $\gamma\pi^+ \rightarrow l^+\nu_l$  ( $l = e, \mu$ ).

enhancement of the neutrino emissivity as well.

### 3 Single neutrino photoproduction on charged pNGB

Photons are also able to produce single neutrinos on charged pNGB:

$$\gamma + \pi^+ \rightarrow l^+ + \nu_l, \quad (3.1)$$

where  $l = e, \mu$ .

The Feynman diagrams contributing to (3.1) are shown in figure 4.

The corresponding matrix element is [22, 23]

$$\mathcal{M} = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c \quad (3.2)$$

with

$$\mathcal{M}_a + \mathcal{M}_b = -ie \frac{G_F}{\sqrt{2}} V_{ud} f_\pi m_l \varepsilon_\mu \bar{u}(p_\nu) (1 + \gamma_5) \left( \frac{p_\pi^\mu}{p_\pi \cdot q} - \frac{2p_l^\mu - \not{q}\gamma^\mu}{2p_l \cdot q} \right) v(p_l), \quad (3.3)$$

$$\begin{aligned} \mathcal{M}_c = ie \frac{G_F}{\sqrt{2}} V_{ud} \varepsilon_\mu \bar{u}(p_\nu) \gamma_\alpha (1 - \gamma_5) v(p_l) \times \\ \times \left[ \frac{F_A}{m_\pi} (-g^{\mu\alpha} p_\pi \cdot q + p_\pi^\mu q^\alpha) + i \frac{F_V}{m_\pi} \epsilon^{\mu\alpha\beta\lambda} q_\beta p_{\pi\lambda} \right], \end{aligned} \quad (3.4)$$

where  $V_{ud}$  is the Cabibbo–Kobayashi–Maskawa matrix element,  $f_\pi$  is the pion decay constant,  $F_A$  is the pion axial-vector form factor.

Let us restrict ourselves to a consideration of the single electron neutrino photoproduction:  $\gamma\pi^+ \rightarrow e^+\nu_e$ . In this case, unless the in-medium mass of the pion becomes very small,  $m_\pi \approx m_e$ , one can safely neglect the contributions of the diagrams (a) and (b) for they are helicity suppressed being proportional to  $m_e$  exactly as in the decay  $\pi^+ \rightarrow e^+\nu_e$  (see eq. (3.3)).

Then, keeping only the contribution of the diagram (c), which is free of the helicity suppression, and squaring (3.2) yields

$$\sum_{\text{spins}} |\mathcal{M}_c|^2 = \frac{\alpha\pi G_F^2}{m_\pi^2} |V_{ud}|^2 s \{t^2 |F_V + F_A|^2 + u^2 |F_V - F_A|^2\} \quad (3.5)$$

so that the corresponding cross section reads

$$\sigma_\pi^c = \frac{\alpha G_F^2}{24 m_\pi^2} |V_{ud}|^2 (|F_V|^2 + |F_A|^2) s^2 \left(1 - \frac{m_\pi^2}{s}\right). \quad (3.6)$$

Making the same assumptions as in section 2 one arrives at the following relation connecting  $Q_{\nu\bar{\nu}}$  with the neutrino emissivity through the process  $\gamma\pi^+ \rightarrow e^+\nu_e$  (denoted by  $Q_\nu$ ):

$$\frac{Q_\nu}{Q_{\nu\bar{\nu}}} = \frac{1}{2} |V_{ud}|^2 \left(1 + \frac{|F_A|^2}{|F_V|^2}\right). \quad (3.7)$$

In (3.7) the factor 1/2 takes into account the fact that the neutrino in the reaction  $\gamma\pi^+ \rightarrow e^+\nu_e$  carries away only a half of the total energy. Numerically, at the vacuum values  $|V_{ud}|^2 = 0.9482$ ,  $F_V = 0.0272$  and  $F_A = 0.0112$  [18], (3.7) gives  $Q_\nu/Q_{\nu\bar{\nu}} = 0.5544$ .

Since the absolute square of the matrix element for the process (3.1) including the contributions of the diagrams (a) and (b) may be useful for similar calculations, its full form is given in the appendix A.

## 4 Conclusions

Photoproduction of neutrino–antineutrino pairs as well as single neutrinos in the reactions  $\gamma\pi^0 \rightarrow \nu_l\bar{\nu}_l$ ,  $\gamma\pi^+ \rightarrow l^+\nu_l$  are studied within the Standard Model. The corresponding cross sections are calculated analytically. All the results of this paper concerning the neutrino photoproduction on charged pions directly applies to the case of the charged kaon target. One has just to perform replacements of the appropriate parameters in the formulae (namely  $m_\pi \rightarrow m_K$ ,  $V_{ud} \rightarrow V_{us}$ ,  $f_\pi \rightarrow f_K$ ,  $F_{V,A} \rightarrow F_{V,A}^K$ ).

These reactions may play a role in thermal evolution of astrophysical objects containing pseudoscalar excitations such as supernovae and compact stars. The energy loss due to neutrino emission in a thermal plasma of photons and pions (kaons) is calculated. It is shown that the neutrino emissivity turns out to be proportional to the total pion decay width. The latter at appropriate pion energies and momenta may grow up to tens of MeV with the density leading thus to a significant enhancement of the neutrino emissivity as well.

The analysis of this paper is also closely related to the problem of pion (kaon) stability in hot media. For example, it is notable that the reaction  $\gamma\pi^0 \rightarrow \nu_l\bar{\nu}_l$  mimics the decay  $\pi^0 \rightarrow \nu_l\bar{\nu}_l$  but in difference from the latter may proceed even at  $m_\nu = 0$  and with considerable probability. In other words, the corresponding cross section does not vanish in the limit of massless neutrinos and observation of neutrino–antineutrino pairs does not therefore require the assumption of Lorentz invariance violation.

The results of the presented analysis of the reactions  $\gamma(\pi^+, K^+) \rightarrow l^+\nu_l$  will be exactly the same for  $\gamma(\pi^-, K^-) \rightarrow l^-\bar{\nu}_l$  if CP is conserved.

## Acknowledgments

This work was supported in part by the Russian Foundation for Basic Research (grant 11-02-12043), by the Program for Basic Research of the Presidium of the Russian Academy of Sciences "Fundamental Properties of Matter and Astrophysics" and by the Federal Target Program of the Ministry of Education and Science of Russian Federation "Research and Development in Top Priority Spheres of Russian Scientific and Technological Complex for 2007-2013" (contract No. 16.518.11.7072).

## A The matrix element squared

The absolute square of the matrix element for the process (3.1) represented by the Feynman diagrams in figure 4:

$$\begin{aligned} \sum_{\text{spins}} |\mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c|^2 &= \alpha\pi G_F^2 |V_{ud}|^2 \times \\ &\times \left\{ 4f_\pi m_l^2 u \left( \frac{f_\pi (2m_l^4 m_\pi^2 - m_l^2 (2m_\pi^4 + (s - m_\pi^2)^2 + 2st) + t(m_\pi^4 + s^2))}{(t - m_l^2)^2 (s - m_\pi^2)^2} - \right. \right. \\ &\quad \left. \left. - \frac{\text{Re}[(F_V + F_A)^*](m_l^2 m_\pi^2 - st) - \text{Re}[(F_V - F_A)^*](m_\pi^2 - m_l^2)(s - m_\pi^2) + su}{m_\pi(t - m_l^2)(s - m_\pi^2)} \right) + \right. \\ &\quad \left. + \frac{1}{m_\pi^2} (|F_V + F_A|^2(m_l^2 - t)(m_l^2 m_\pi^2 - st) - |F_V - F_A|^2 u(m_l^2(s - m_\pi^2) - su)) \right\}. \quad (\text{A.1}) \end{aligned}$$

One can see that (A.1) in the limit  $m_l = 0$  is reduced to (3.5).

## References

- [1] Y. Nambu, *Quasiparticles and gauge invariance in the theory of superconductivity*, *Phys. Rev.* **117** (1960) 648.
- [2] J. Goldstone, *Field theories with "superconductor" solutions*, *Nuovo Cim.* **19** (1961) 154.
- [3] J. Goldstone, A. Salam, S. Weinberg, *Broken symmetries*, *Phys. Rev.* **127** (1962) 965.
- [4] Y. Nambu, G. Jona-Lasinio, *Dynamical model of elementary particles based on an analogy with superconductivity. I*, *Phys. Rev.* **122** (1961) 345.
- [5] Y. Nambu, G. Jona-Lasinio, *Dynamical model of elementary particles based on an analogy with superconductivity. II*, *Phys. Rev.* **124** (1961) 246.
- [6] S. Nussinov, R. Shrock, *On the  $\pi$  and  $K$  as  $q\bar{q}$  bound states and approximate Nambu–Goldstone bosons*, *Phys. Rev. D* **79** (2009) 016005 [arXiv:0811.3404].
- [7] J.N. Bahcall, R.A. Wolf, *Neutron stars. I. Properties at absolute zero temperature*, *Phys. Rev.* **140** (1965) B1445.
- [8] O. Maxwell et al., *Beta decay of pion condensates as a cooling mechanism for neutron stars*, *ApJ* **216** (1977) 77.
- [9] H. Umeda et al., *Neutron star cooling and pion condensation*, *ApJ* **431** (1994) 309.
- [10] P. Jaikumar, M. Prakash, T. Schaefer, *Neutrino emission from Goldstone modes in dense quark matter*, *Phys. Rev. D* **66** (2002) 063003 [astro-ph/0203088].
- [11] F. Arretche, A. A. Natale, D. N. Voskresensky, *Medium effects in the pion-pole mechanism ( $\gamma\gamma \rightarrow \pi^0 \rightarrow \nu_R \bar{\nu}_L (\nu_L \bar{\nu}_R)$ ) of neutron star cooling*, *Phys. Rev. C* **68** (2003) 035807 [astro-ph/0208362].
- [12] S. Reddy, M. Sadzikowski, M. Tachibana, *Neutrino rates in color flavor locked quark matter*, *Nucl. Phys. A* **714** (2003) 337 [nucl-th/0203011].
- [13] M. Loewe, C. Villavicencio, *Pion stability in a hot dense media*, arXiv:1107.3859.
- [14] S. S. Gershtein, Yu. Ya. Komachenko, M. Yu. Khlopov, *Single photon production in exclusive neutrino-nucleon reactions*, *Sov. J. Nucl. Phys.* **33** (1981) 860.
- [15] Yu. Ya. Komachenko, M. Yu. Khlopov, *Weak-electromagnetic decays of neutral pseudoscalar mesons* *Sov. J. Nucl. Phys.* **46** (1987) 679.

- [16] J. A. Harvey, Ch. T. Hill, R. J. Hill, *Anomaly mediated neutrino-photon interactions at finite baryon density*, *Phys. Rev. Lett.* **99** (2007) 261601 [arXiv:0708.1281].
- [17] I. Alikhanov, *Exclusive production of pseudoscalar mesons in neutrino-photon interactions*, *Phys. Lett. B* **706** (2012) 423 [arXiv:1109.1261].
- [18] C. H. Chen, C. Q. Geng, C. C. Lih, *Study of the radiative pion decay*, *Phys. Rev. D* **83** (2011) 074001 [arXiv:1006.2939].
- [19] V. G. Vaks, B. L. Ioffe, *On  $\pi \rightarrow e + \nu + \gamma$  decay*, *Nuovo Cim.* **10** (1958) 342.
- [20] V. F. Muller, *On the connection between the decays  $\pi^0 \rightarrow 2\gamma$  and  $\pi^+ \rightarrow e^+ + \nu + \gamma$* , *Z. Phys.* **173** (1963) 438.
- [21] G. Raffelt, D. Seckel, *Multiple-scattering suppression of the bremsstrahlung emission of neutrinos and axions in supernovae*, *Phys. Rev. Lett.* **67** (1991) 2605.
- [22] D. A. Bryman, P. Depommier, C. Leroy,  *$\pi \rightarrow e\nu$ ,  $\pi \rightarrow e\nu\gamma$  decays and related processes*, *Phys. Rep.* **88** (1982) 151.
- [23] C. H. Chen, C. Q. Geng, C. C. Lih, *T-violating muon polarization in  $K^+ \rightarrow \mu^+ \nu \gamma$* , *Phys. Rev. D* **56** (1997) 6856 [hep-ph/9610533].